Problem Solving 6

Lecture 17 May 02, 2021

- Q1 (leftover from last week). Suppose $\mathrm{a}_1, \mathrm{a}_2, \ldots, \mathrm{a}_5$ are a permutation of 1,2,…,5. What is the max possible value of

$$a_1a_2 + a_2a_3 + \cdots + a_5a_1$$

- (Try to solve the problem for arbitrary n instead of 5).

- Q2. The point P is moving on the points of the xy-plane with integer coefficients. At any moment, if P is at (a, b), depending on the value of $a + b \ \ mod\ 4$ (which can be 0,1,2,3) it will move in the following way:
- $0 \ \rightarrow right$
- $0 \ \rightarrow up$
- $0 \ \rightarrow left$
- $0 \ \rightarrow down$

For how many different starting points, after 100 moves, P will be at (0,10)?

- Q3. Consider a triangle ABC. Let D be a point on AB and E be a point on AC. Let P be the intersection point of BE and CD. If the areas of ADE, BPD, and CEP are 5, 8, and 3, respectively, then what is the area of ABC?

- Q4. How many numbers within the set $\{1,2,\dots,1999\}$ are difference of two squares (i.e. $n = a^2 - b^2$)

- Q5. C_1 and C_2 are two circles of radii 3 and 1, respectively, such that the distance between their centers is 10. Let S be the set of points in the plane that are the middle point of a line segment with one end on C_1 and the other end on C_2.

What is the area of S?

- Q6. Consider the sequence defined by $a_{n+1} = 2a_n + 5$ and $a_1 = 1$.
- Which of the following numbers will appear in this sequence

- 562301
- 786427
- 16485
- 3123
- 51519

- Q7. In a factory, every worker is friend with some other workers and his/her salary is equal to the average of the salary of his/her friends. Which of the followings is correct:

- There is one worker whose salary is equal to the average of the salary of all others.
- No one earns more than twice of any other person.
- All the salaries are the same
- Every two friends earn the same
- None

- Q8. Suppose x, y, z are 3 real numbers such that $xyz(x + y + z) = 1$.
- What is the minimum possible value of $(x + y)(y + z)$?